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The small, insanely competitive women's figure skating world is still seething over an incident at the 1995 World Championship. At one point in the competition, the top three skaters were Chen Lu of China (first place), Nicole Bobek of the United States (second), and Surya BonaJy of France (third). All three had completed their performances, the judges had scored them, and the media had reported their preliminary rankings. Then fourteen-year-old American Michelle Kwan took the ice. When Kwan's performance was factored in, she came in fourth place, The odd thing is, Kwan caused Bobek's and Bonaly's standings to flip. Bobek dropped to third place and went home with a bronze medaL Bonaly rose to second place and took home the silver.

In other words, before Kwan skated, Bobek was judged a better skater than Bonaly. Afterward, Bonaly was a better skater than Bobek, according to the same judges. No judge had changed his or her mind about the two skaters; their votes for them had already been cast.

You may think there must have been something funny about the formula used to combine the judges' rankings. Trust me-there wasn't, If I explained the whole voting system, you would nod your head and say, *That sounds fair*.

During the men's competition of the 1997 European Championship, nearly the same thing happened. This time the top three skaters were Alexei Urmanov, Viacheslav Zagorodniuk, and Philippe Candeloro, in that order. The final contestant, Andrejs Vlascenko, came in last *ofsix*. Adding in the votes for Vlascenko caused Candeloro to move up to second place. Candeloro went home with a silver medal because the judges hadn't liked the way Vlascenko skated. Meanwhile, Zagorodniuk dropped to third place, forfeiting a silver medal because Vlascenko came in last.

This "Great Flip-Flop" touched off a firestorm in the skating world. Immediately afterward, International Skating Union (ISU) chairperson Ottavio Cinquanta admitted that something was wrong with the judging system. He vowed to fix it. Skating columnist Sandra Loosemoore disagreed. She felt the old system was okay and that the ISU should be more concerned with educating the public about these possible flip-flops. Loosemoore suggested including a printed copy of the scoring method in skating program booklets, incorporating announcements about **it** on the public address system, and providing a "technical liaison" to explain scoring to the media. She proposed that flip-flops be viewed as "something that adds significantly to the suspense of the event!"

The ISU nonetheless rolled out a new judging system in 1998. Under the system, Cinquanta promised, "If you are in front of me, you will remain in front of me!"

He was wrong. Cinquanta apparently had never heard of the impossibility theorem. If he had, he would have known that he was trying to do exactly what Kenneth Arrow had proved impossible. Loosemoore quickly provided an example of a case where the ISD's new system would fail miserably, and she cited a statistical analysis claiming that the new system was actually *worse* than the old one.

There are many situations where everyone can't have his or her own way. A group must arrive at a choice that is binding on all of its individual members. What is the best way to do this? Political philosophers had been asking that question for ages. Arrow's novel approach was to recast it as a question in pure logic. He observed that social choices generally follow a set of rules-a constitution, a parliamentary procedure, or a cultural tradition. These rules are legalistically precise to minimize the possibility of dispute.

In Arrow's terminology, any system for devising a social choice is caBed a *constitution*. You can compare Arrow's constitutions to voting machine software. The constitutions take the voters' ballot markings as input. They tally the votes according to a precise, step-by-step algorithm. Then they output the winners,

A constitution does not have to be fair or democratic or even reasonable. It could be like a hacker's rigged voting software, slanting the election in favor of a desired candidate. Of course, the important question is how to devise a constitution that is fair and logical. This is what Arrow tried to do-until he discovered it couldn't be done.

Americans are so used to "one person, one vote" that they often imagine this is the only sensible way to vote. It's not. (In fact, we'll see that it's about the *least* sensible way to vote!) "One person, one vote" is known as a plurality vote because the winner is the candidate who receives the largest number of votes. Many other voting methods, such as instant-runoff voting, use ranked ballots. Instead of just marking your favorite candidate with an X, you mark that candidate 1, and then mark your second-place choice 2, your third-place choice 3, and so on. This ranked, or preferential, ballot is used throughout the world, though rarely in the United States.

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Plurality Ballot	Ranked Ballo
D Adams	2 Adams
D Bush	3 Bush
🗙 CI,y	Clay
D DeWitt	DeWitt

Arrow's proof assumes a ranked ballot. In so doing, it includes the familiar plurality vote as a special case: a plurality vote constitution simply says to ignore all but the first-place choices. Arrow's analysis therefore covers virtually every voting system used in elections throughout the free world, plus systems not even imagined today.

One important feature of voting is *not* included in Arrow's analysis, though, This is strategic voting. Sometimes people intentionally vote for someone other than their true favorite, An example is a supporter of a third-party candidate who votes for the more acceptable of the two major-party candidates. Such a voter is pretending to prefer the major-party candidate because she doesn't want to "throwaway" her vote on a favorite who has no chance of winning.

Arrow assumes complete honesty on the part of the voters, You might imagine that, instead of ballots, a polygraph test verifies each voter's sincere preferences about the candidates. Could you use this flawless knowledge about voter preferences to devise a perfect voting system? This is essentially what Arrow asked. The answer is no,

In order for Arrow to make a compelling case, it was necessary to set some specific conditions. He began with the claim that any reasonable way of voting (constitution) must meet a set of commonsense conditions, In Arrow's original proof there are five conditions, I will give a simplified (but equivalent) version with just four. One condition is *transitivity*. "If a man says he likes Republicans better than Democrats and Democrats better than Communists, then we think he is strange indeed if he also says he likes Communists better than Republicans," as political scientist William H. Riker once put it. Arrow stipulates that every voter must have transitive preferences. Someone who prefers a Republican to a Democrat and the Democrat to a Communist *has* to prefer the Republican to the Communist.

We already know that there are cases where two-way votes of transitive *voters* can produce an intransitive outcome. Arrow therefore demands transitivity of the election's outcome, too. A useful voting system has to be decisive and identify one unambiguous winner. Arrow further requires that the outcome be a full ranking of all the candidates. We want to know not only who won but also who came in second, third, and so on.

A second condition is that the voting system must respect *unanimity*. In the unlikely event that *every single voter* prefers candidate Adams to Buchanan, then Adams should beat Buchanan in the final tally. Any other result is ridiculous.

You may wonder why we need this condition when unanimity is unheard of in a real election with thousands or millions of voters. Wouldn't it make more sense to demand that when a substantial *majority* of the voters prefer Adams to Buchanan, Adams is preferred in the outcome?

Notice that any voting system that always respects the desires of a "substantial majority" will also respect unanimous desires. Unanimity is actually the *least* demanding condition we can ask for. This keeps Arrow's result as far-reaching as possible.

A third condition is a no-brainer: *nondictatorship*. Suppose that whenever there's a big decision to be made, every citizen goes to the polls and marks a ballot. It's all a pretense. Only one vote counts, and that's Joe Stalin's. This is a dictatorship (even if everyone does go through the motions of voting). The nondictatorship condition says,

reasonably enough, that there must not be a dictator-voter who always gets his way, no matter how everyone else votes.

Arrow had to include the nondictatorship condition because a dictatorship-hardly a fair voting system----can easily obey his other conditions. Assuming that Joe Stalin knows his mind, the above system is transitive. Also, our dictatorship never violates the unanimous will of the people. When *everyone* (including Joe Stalin) votes for increasing the vodka ration, more vodka it is. Of course, when everyone *except* Joe Stalin votes for vodka, it's another story.

Arrow had one more condition. He gave it a wordy label: *independence* of irrelevant alternatives. It is best introduced with a few examples. One of the most memorable is due to the Columbia University philosopher Sidney Morgenbesser 0921-2004), a specimen of a particularly rare brand of genius, better remembered for his wit than for anything he published. According to the story, Morgenbesser was in a New York diner ordering dessert. The waitress told him he had two choices, apple pie and blueberry pie. "Apple," Morgenbesser said.

A few minutes later the waitress came back and told him, oh yes, they also have cherry pie.

"In that case," said Morgenbesser, 'Til have the blueberry."

Cherry pie is what Arrow calls an "irrelevant alternative." It is irrelevant because, given the chance to order cherry pie, Morgenbesser rejected it. But something you don't want anyway shouldn't cause you to change what you *do* want. When the two options were apple and blueberry, Morgenbesser preferred apple. Fine. Then cherry was added to the menu. It would have made perfect sense for Morgenbesser to switch his order to cherry. It would have made sense for him to stick with apple. Switching to blueberry was crazy!

The Great Flip-Flop is an example of a vote failing to meet independence of irrelevant alternatives. The gold- and silver-medal winners should not flip because of a skater who came in sixth. So Arrow proposed, as a commonsense condition, that whether candidate A beats B should *not* depend on any "irrelevant" third-party C. To give one final example, the obvious one, whether AI Gore beats George W. Bush should not depend on whether Ralph Nader is in the race. As both the figure-skating and the political examples show, people can get pretty upset when this condition is violated.

You may not feel that this and the other conditions say everything there is to say about fair elections. That's okay. Arrow's conditions, like Euclid's axioms, were purposely kept minimal. The important thing is that all are absolutely necessary preconditions of any reasonable democratic system. Arrow derived a conclusion whose shock waves are still being felt. He proved that it is impossible to design a voting system meeting these commonsense conditions. It is like squaring the circle or designing a perpetual-motion machine. *It can't be done*. Consequently, *any* voting system of the broad class described by Arrow has serious problems.

Arrow called this result an impossibility theorem, But "when I used that term at the Cowles Commission, the director, Tjalling Koopmans, felt that it was too pessimistic." On Koopman's advice, Arrow switched to the more upbeat name "general possibility theorem" in his dissertation. Afterward, he and almost everyone else reverted to the more accurate-and dismal-name.

The audacious nihilism of Arrow's proof was quickly compared to Coders. "The search of the great minds of recorded history for the perfect democracy, it turns out, is the search for a chimera, for logical self-contradiction," wrote MIT economist Paul Samuelson in 1952. "New scholars all over the world-in mathematics, politics, philosophy, and economics-are trying to salvage what can be salvaged from Arrow's devastating discovery that is to mathematical politics what Kurt Gödel's 1931 impossibility-of-proving-consistency theorem is to mathematical logic."

No century before the twentieth could have been shocked by the revelation that the social contract is founded on as flimsy a foundation as mathematics itself. It was partly for this work that Arrow won the 1972 Nobel Prize in Economics (shared with John R. Hicks) "for their pioneering contributions to general economic equilibrium theory and welfare theory." The Nobel committee's press release identified the impossibility theorem as "perhaps the most important of Arrow's contributions to welfare theory." Arrow himself rates the theorem as his most important achievement, and it has been cited more than any of the other papers he has published in a long, diverse career as an economist.

Like Michelangelo's Sistine Chapel, the impossibility theorem was so breathtakingly original that it made everyone else's work in voting theory look old-fashioned. A couple of academic careers were probably s[Unted as a result. Abram Bergson, whose work had been the jumpingoff point for Arrow's, sat on the panel quizzing Arrow for his doctorate. "Obviously, he was a little miffed," Arrow recalls, "but very fair." Some conjecture that Arrow ultimately cost Bergson a Nobel Prize.

A sadder case was the perpetually underappreciated Duncan Black. Black claimed that he discovered the paradox of voting in 1942. Not until November 1949, however, did he and R. A. Newing submit a paper to *Econometrica* describing the paradox. It took eighteen months for the journal to accept it. The editor insisted that Black cite Arrow's paper (which went far beyond Black's). Feeling cheated of priority, Black withdrew the paper and had it printed privately as a bookJet titled *Committee Decisions with Complementary Valuation*. Few ever saw it. Twenty copies remained in Black's house at his death. Black himself must have written the jacket copy: "Whatever the merits or demerits of the book, it can safely be said that there is no other which has attempted to deal with this subject."

Black was not entirely out of the loop. In December 1948 the RAND Corporation's Joseph Goldsen wrote Black that "a group of

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American mathematicians and political scientists" were interested in his work and would appreciate receiving offprints. Black had never heard of the RAND Corporation. He checked it out with the British Consul in San Francisco, An official informed him that "the activities of the Rand Corporation are highly classified" and the "United States Air Force would much prefer that, if you decided to respond to Mr. Goldsen's enquiry, it should be communicated to the Corporation through themselves." Black must have decided he wanted nothing to do with the RAND Corporation and its highly classified activities. He never replied to Goldsen.

For the past half century scholars and journalists have struggled to understand what the impossibility theorem means. It is possible to find in the literature counterparts to Elisabeth Kubler-Ross's stages of denial, anger, bargaining, depression, and acceptance. Kubler-Ross was characterizing reactions to impending death—and as some saw it, Arrow's work was a death knell for any idealistic notion of democracy. It has struck many as a portentous comment on the human condition. We can't connect, we can't find consensus, we can't get along.

"During the 1930s and 1940s there was a pervasive sense of dismay and defeat among the intellectuals of the West over what they took to be the inevitability of the triumph, both external and internal, of fascist or communist alternatives to democratic capitalism," historian S. M, Amadae wrote in *Rationalizing Capital-ist Democracy* (2003). Authoritarian states were moving toward centrally planned economies. This was the wave of the future, and it worked (or so Lincoln Steffens said). In the UllileJ States there was a new eagerness Lu justify previously unquestioned Western values. Could the uncertain workings of democracy and individual choice compete with scientifically planned economies)

The RAND Corporation was a focus of America's collectivist panic,

Its creation was an attempt to beat the Soviet planners at their own game(s), to show that reasoned, scientific policy decisions could be compatible with American democracy. This meant, in other words, that RAND's elite would devise ways to make sound decisions on behalf of the American masses. Amadae has termed this doctrine "rational choice" and wrote that "it is no exaggeration to say that virtually all the roads to rational choice theory lead from RAND." Amadae interprets Arrow's theorem as "part of the campaign to reassert the tenets of governmental rule legitimized by popular consent, but not susceptible to fascist or authoritarian perversions."

Mathematics is no respecter of ideology. What Arrow ended up proving did not exactly help the case for democracy. Perhaps his theorem resonated all the more deeply for this reason. It articulated a central anxiety of the age, that democracy and individualism might in some deep way be inadequate against the collectivist, authoritarian alternative.

Social Choice and Individual Values was published cluring the peak of Joseph McCarthy's communist witch hunt. In a climate already suspicious of intellectuals who criticized the American system, Arrow's work was easily miscast. "I gave a talk on this at the December 1948 meeting of the Econometrics Society," Arrow said. 'There was a fellow there, a Canadian economist named David McCord Wright. He said that he hadn't seen the value of *freedom* mentioned anywhere in this. As he was going out, he said to a mathematician who was interested in economics, a fellow named Kenneth May: 'Oh! Arrow and Klein are communists!''' Lawrence Klein was the meeting's chainnan. Arrow was more amused than offended because Wright must not have known May. "May was a realleftie," Arrow says. "He probably *was* a communist."

Among those who attended to what Arrow was saying, the effect was more to reconcile people to the status quo than to make them want to tear it down. One reaction to Arrow's theorem was that it

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echoed the oft-quoted wisdom of Winston Churchill in a 1947 House of Commons speech: "Many forms of Government have been tried, and will be tried in this world of sin and woe. No one pretends that democracy is perfect or all-wise. Indeed, it has been said that democracy is the worst form of government except all those other forms that have been tried from time to time:'

That was the optimistic view. To look at it another way, Arrow's theorem says that election outcomes can be decided by quirks of procedure as much as the voters' authentic wishes. That better recalls a remark, probably apocryphal, attributed to Joseph Stalin: 'Those who cast the votes decide nothing. Those who count the votes decide everything,"

Upon the 1972 announcement of Arrow's Nobel Prize, Paul Samuelson supplied the now-standard journalist's gloss: "What Kenneth Arrow proved once and for all is that there cannot possibly be ... an ideal voting scheme,"

To some extent, Arrow's theorem refutes the notion of a "will of the people." We all believe in a public will, envisioning it in our own political image-that is to say, as decisive and self-consistent. This belief makes it easy to be optimistic about democracy. Voting is then a way of channeling the public will. The many different ways of designing ballots and tallying votes can be thought of as competing road maps of the same region. Though different in superficial ways, all show the same tenitory. Because of that, it shouldn't matter which map (voting system) we use. All are going to get us where we need to go.

We all know that the map is not the territory. What if there were no territory--only maps? Arrow's theorem says that there are situations where the "will of the people" is ill-defined, where rational people are collectively irrational. A decisive voting system will come up with a winner, but that winner may differ from the winner decided under another voting system that *also* sounds fair and reasonable. There may be no unique democratic outcome.

Is there any way around this? Arrow's result applies only when there are three or more candidates. A simple majority vote between two candidates is as fair as anyone could desire.

Much voting *is* between two alternatives. There are yea-or-nay votes for parliamentary motions and yes-or-nos for ballot referenda. In elections for office, America's two-party system often provides a de facto binary choice.

The role of the two-party system in avoiding electoral paradox was recognized long before Arrow. In an 1885 book, *Congressional Government*, future president Woodrow Wilson argued that the two-party system must present voters with clear-cut, either-or choices. Wilson was an Anglophile taking as his model Victorian England. Writing at a certain remove from the rough-and-tumble of British politics, he believed that the tightly disciplined British parties offered voters a clear philosophical choice lacking in America. Why can't the Democrats and Republicans be more like the Tory and Labour parties? Wilson wanted to know.

One thing Wilson did *not* know was that he would one day become president only because the two-party system *failed*. In 1912 ex-president Teddy Roosevelt's independent candidacy split the Republican vote, leading to Wilson's victory. Had there been a single Republican running, Wilson almost certainly would have lost.

Upon closer inspection, two-way choices are the exception in politics. There are always more than two people wanting to run for an important office. Most of them bow out when they fail to gain their party's nomination. Every bill or referendum put to the vote is one out of the infinity of bills or referenda that potentially could have been proposed. When we rank-and-file voters have a two-way choice, it's because someone more powerful has decided what that two-way choice is going to be. Harvard economist Amartya Sen aptly called Arrow's theorem "the big bang:' From out of nothingness, it created the ever-expanding universe of social choice theory. Scholars quickly set about finding ways to interpret and build upon Arrow's work. I will have more to say later about the ever-morphing perceptions of the impossibility theorem. For the moment, let me add this caveat: *The devil is in the details*. I have defined Arrow's conditions informally. His result is not a philosophical thesis but a demonstration in pure logic, contingent on the precise formulation given in his paper. Arrow himself made a slight error in his original publication, discovered by Julian Blau in 1957 and corrected in the second edition of *Social Choice and Individual Values*. This underscores the far greater hazard of reducing Arrow's theorem to a maxim such as "no voting system is perfect:'

Oskar Morgenstern was dying. An aggressive spinal cancer racked his body and sapped his spirit. As his own health ebbed, Morgenstern was attempting to look after Kurt Codel, who was also dying-and was almost out of his mind. The logician had lately become convinced that he was being poisoned. He hinted that one doctor was prescribing a poisonous drug. He asked another doctor whether he was an impostor impersonating the "real" doctor.

Morgenstern asked one of Coders physicians for an opinion. Was Gödel a danger to himself or others? The doctor thought he was okay, if only he'd eat. Gödel ate so little that his weight had diminished to sixty pounds.

In fact, Gödel would survive Morgenstern.

Sometime in 1976, Morgenstern received a paper from an acquaintance, a mathematically trained magazine editor named George A. W. Boehm. People were always sending Morgenstern papers, hoping he would vet them or convey them to an even more distinguished authority. Boehm's paper described a novel voting system. It purported to prevent the spoiler effect-to encourage honest voting-and perhaps even to keep the Hiders of the world from power over majorities who disliked them.

The amazing thing about the system was how *simple* it was. A single mimeographed sheet described it in full.

Morgenstern filed away Boehm's ingenious scheme. He was too sick a man to do much with it.