ONE

Game Theory

Kurt Giidel, the most brilliant logician of the twentieth century, had no interest in politics. He showed no apparent alarm when Hitler became chancellor of Germany. (Codel closed a 1936 letter with a cordial "Heil Hitler:') He was equally unconcerned when Hitler annexed Austria in 1938. Then, in August 1939, war began. Things quickly got worse in Gödel's Vienna, In November, Gödel was attacked by a gang of Nazi youths. He was not Jewish, but people thought he looked Jewish, or scholarly, or cosmopolitan. Gödel was in the company of his less scholarly girlfriend, Adele Porkert, who worked in a disreputable nightclub. She fought off the Nazis with her umbrella.

Gödel was soon drafted. As he had no intention of fighting, he and Porkert, now married, fled the country. Gödel had a visa and an open invitation to work at the Institute for Advanced Study in Princeton, New Jersey. As things worked out, he and Porkert would spend the rest of their lives in Princeton. The years rolled by. In 1947 Gödel decided it was time to apply for American citizenship. He needed two American citizens as witnesses. Two of his best Friends volunteered. They were Albert Einstein and Oskar Morgenstern (an economist). Like all immigrants, Gödel was supposed to study up on the American system of government. He threw himself into the task. Apparently for the first time in his life, he became interested in the political process.

The day before the exam, he informed Morgenstern that he had uncovered a logical contradiction in the **U.S.** Constitution. Morgenstern thought this was amusing-until he realized how serious Gödel was about it.

Gödel was famous for discovering a logical contradiction in mathematics. Ever since Euclid, mathematicians had aspired to put logic and math into tidy packages. A set of unquestioned axioms would be given. From those axioms it would be possible to prove all true mathematical statements and to disprove all falsehoods. It would also (presumably) be possible to prove the system's consistency. If it is possible to show that "2 + 2 = 4" is true (as it should bel, then it must be impossible to prove that the Same statement is false.

This goal seemed reasonable to almost everyone. Then, in 1931, Gödel shattered the millennia-old dream. He demonstrated that no valid logical system can prove itself to be free from contradiction. The gist of Godel's proof might be rendered like this: Anyone who says he *always* tells the truth is lying. Gödel showed that this statement holds not only for used-car salespeople and politicians but also for the most abstract constructions of logic.

It was this work that brought Gödel renown and led to his esteemed position at the Institute. Morgenstern confided to Einstein that he was worried that Gödel would launch into a rant about the "contradiction" he had discovered in the Constitution during his citizenship exam. The examiner might deny Gödel citizenship just for that. Einstein agreed that they had to make sure this didn't happen.

The citizenship exam was scheduled for December 5, 1947, in

Trenton. Since Adele did not permit Codel to drive anymore-he had once fallen into such a deep meditation that he forgot he was, at that moment, driving a car-Morgenstern volunteered to drive him. When he picked up Einstein, the physicist got in and turned to Gode!. "Well, are you ready for your next-to-last test?"

"What do you mean 'next to last'?" Gödel inquired.

"Very simple," Einstein answered. "The last will be when you step into your grave."

Einstein could have a morbid sense of humor. But, according to plan, he kept Godel occupied during the drive. When they got to Trenton, Einstein recognized the judge, Philip Fonnan, as the one who had administered his own oath of citizenship. Forman pulled Einstein and friends out of line and took them into his chambers. The judge and Einstein chatted while Godel sat silently. Forman remarked how wise Godel was to leave Cennany and its "evil dictator." "Do you think a dictatorship like that in Germany could ever arise in the United States?" Forman asked.

"I know how that can happen!" Codel said, and he began his explanation. To Morgenstern's and Einstein's relief, Fonnan cut him off, telling him he didn't need to go into all that.

Godel passed the exam. He returned to Trenton on April 2 to take the oath of citizenship. At the ceremony, Judge Forman gave a patriotic speech about American values. It was probably a talk he had given many times. Godel found himself moved by it (as he recounted in a letter to his mother). He went home feeling that American citizenship was something special and fine. Godel could switch unpredictably from cold logic to maudlin sentiment, and it was hard for others to tell what would set him off. He adored the Disney film Bambi. He saw Snow White at least three times.

The "flaw" that Codel found in the U.S. Constitution was in Article V, the one that provides for amendments. It begins:

Article V.

The Congress, whenever two thirds of both Houses shall deem it necessary, shall propose Amendments to this Constitution, or, on the Application of the Legislatures of two thirds of the several States, shall call a Convention for proposing Amendments, which, in either Case, shall be valid to all Intents and Purposes, as Part of this Constitution, when ratified by the Legislatures of three fourths of the several States, or by Conventions in three fourths thereof, as the one or the other Mode of Ratification may be proposed by the Congress.

Godellooked at the Constitution as if it were a set of axioms. Just as the ideal mathematical system should be able to derive any true statement from its axioms, the ideal governmental system should permit any good and equitable constitution to be derived from the original one, by orderly process of amendment. Still, we presumably do not want a Constitution that can amend itself into Nazi Germany, Orwell's 1984, or some other kind of dystopia.

This is where Article V fails, Gödel felt. By *permittingeperything*, it guarantees *nothing*. In principle, the Bill of Rights could be rescinded by a future amendment-just as Prohibition (the Eighteenth Amendment) was repealed by the Twenty-first Amendment. It may be supposed that a two-thirds majority of both houses would never countenance a major erosion of individual liberties, Article V could amend itself. In theory, two-thirds of Congress could vote in a new amendment saying that only a simple majority is required to amend the Constitution, The smaller the threshold, the more likely it is that a strongly motivated faction might manage to pass an amendment that many find unconscionable,

In the 19405 many Americans felt superiority over the totalitarianism existing in Germany, Italy, and the Soviet Union, Wartime rhetoric implied that America had a patent on democracy. Gödel found this "it can't happen here" attitude unconvincing, In 1932 Adolf Hitler ran in a democratic election for president of Germany. He got 30.1 percent of the vote. That put him a distant second behind Field Marshall Paul von Hindenburg. Under the German system, the first election was followed by a runoff between the top three candidates. Hitler did only modestly better in the runoff, capturing 36.8 percent of the vote. Hindenburg won with 53.0 percent.

'We're beaten; terrible outlook," wrote Hitler's political consultant-Joseph Goebbels. Goebbels could not have guessed how quickly Hitler's fortunes would turn. On January 30, 1933, President Hindenburg appointed Hitler chancellor. Neither of his first two appointees had worked out, and Hindenburg may have felt he was running out of viable candidates.

Less than a month later, communist terrorists burned the Reichstag building. They may have had operational support from the Nazis. The nation's factions put aside their differences to pull together in the crisis. The Reichstag considered a bill to suspend the constitution and give Hitler temporary dictatorial powers. It was not necessarily a crazy idea. The argument was that an enlightened dictator could deal with the crisis better and more quickly than a slow-moving legislative body. The motion passed 441 to 84. It was the first majority Hitler ever got, and the last he would ever need.

It was of course Hitler who was ultimately responsible for Godel, Einstein, and Morgenstern's trip to the citizenship hearing. None would have been in America otherwise.

After Einstein's death, in 1955, Gödel found himself socially isolated. Morgenstern was perhaps the only real friend he had left in the world. Gödel succumbed to a set of neuroses that made him the talk of Princeton. He feared germs, and wandered the streets in a ski mask to avoid contagion. He spent his last years working on a mathematical proof of the existence of God. He came to believe in ghosts, demons,



"Goebbels, Goebbels, give me back ml millions," II, tler says 10 hIS tearful propaganda minister in John Heartfi...Id's 1935 photomontage Goebbels's staff had been accused of embezzling millions, and Hitler's unpopular policies cost him millions of supporters. Hitler epitomizes I troubling paradox I politician adored by I plutal'I} mal be detested by a majority. (George Eastman House

tele pathy. life afler death, and time travel- the latter based on his own solutions to Einstein's field equations.

Spending time in the company of Gödel made Oskar Morgenstern look almost normal in comparison, Morgenstern 100 spent his life never quite fitting into any social context. Like Codel, he had been part of the glittering circle of intellectual luminaries in prewar Vienna. Morgenstern's diaries, now at Duke University, bristle with anti-Semitism. He did not fit in with the Nazis, either. He fled to America, spending much of the remainder of his life in the company of brilliant Jews. At Princeton, he reinvented himself as a man of the world, a connoisseur of wine, women, music, and art. He told people he was an illegitimate descendant of Kaiser Friedrich III. To much of the Princeton University community, however, he was a vain, petty, humorless status-seeker. His stiff Old-World manner inspired his students to tag him *Herr Professor*. Morgenstern once introduced the four younger people working with him---two of them had Ph.D.s, and two were working toward their doctorates----to a European visitor as Dr. Whitin, Dr. Shubik, "und zwei Studenten [and two students]." He couldn't bother to give two names. Thereafter, "zwei Studenten" became an office punch line.

Whatever his social deficiencies, Morgenstern played a vital role in the community of science. He was a cross-pollinator. When he came across a new and important finding, he would put his own work aside and promote the new idea like a stage mother, nagging people into paying attention. The idea's originator would often be too busy or too unworldly to do so.

Had the muse of genius allotted Morgenstern a steadier flow of great ideas, he might have lacked the time to play this role. A man who cared more about being liked could not have deployed sharp elbows as effectively as he did.

The most impressive of Morgenstern's projects was game theory, the creation mainly of Hungarian-born mathematician John von Neumann. Despite the name, game theory is not primarily about games such as chess or Monopoly or Halo. It is more an exact science of strategy. It explores how rational adversaries make decisions, knowing that their opponents are trying to second-guess or double-cross them. In 1928 von Neumann published the paper inaugurating this field. Like

everything else von Neumann did, it was considered brilliant. Then von Neumann moved on to other things.

Morgenstern believed that game theory had important applications to economics. When he came to America, he choose Princeton University over other schools because he wanted to be near von Neumann, one of the true geniuses who worked at the nearby Institute for Advanced Study. There was something almost stalker-like in this move, for Morgenstern had never met von Neumann.

In February 1939, von Neumann attended a lecture Morgenstern gave on business cycles. After the talk Morgenstern cornered von Neumann and [old him he was thinking of writing a paper on game theory's applications to economics. Von Neumann said he'd be glad to read it for comments.

Morgenstern showed several drafts to von Neumann. The mathematician tactfully said they needed a little more polish. Von Neumann suggested a collaboration.

With two authors, the "article" grew. When it had become too long to publish in a journal, the two approached Princeton University Press about putting out a slim volume of about a hundred pages. The press agreed. When the authors delivered the typescript at the start of 1943, it came to twelve hundred pages.

The book appeared in 1944 under the title *Theory of Games and Economic Behavior*. Von Neumann generously proposed that the names of the two authors be listed alphabetically. Morgenstern insisted that von Neumann's name go first.

"The skepticism concerning Morgenstern's contribution to the theory of games was widespread," conceded Martin Shubik, one of the most sympathetic of Morgenstern's students. At Princeton, it was generally understood that, for all of his schmoozing with von Neumann, Code!, and Einstein, Morgenstern was emphatically *not* on their level. He would sometimes sit in on mathematical seminars and ask questions that appeared to confirm this assessment.

Shubik tells of an excruciating lecture in which Morgenstern spent *three hours* trying and failing to reproduce a result from "his" game theory book. "We would have all been happier," Shubik said, "if Oskar had not attempted to go through formal proofs."

John von Neumann had his talking point down pat.

- **a.** Johnny, what did Morgenstern *really* contribute? Come on. You can tell.
- A. "Without Oskar, I would have never written the *Theory of Games and Economic Behavior.*"

No politician could have handled the question better.

I met Kenneth Arrow on a sunny afternoon at the Stanford Faculty Club. At age eighty-four, he was vigorous and unpretentious enough to arrive for lunch by bicycle. With his helmet checked and lunch in front of us, I began my list of prepared questions. One was about Oskar Morgenstern. At my mention of the name, Arrow winced. "} never knew how well he understood game theory," he said quietly. It had been more than fifty years since Arrow had met Morgenstern. That Morgenstern's memory still had that astringent power says something about the man.

Arrow came to meet Morgenstern through a complex chain of events. Kenneth Joseph Arrow was born in New York City on August 23, 1921, the son of Harry and Lillian Arrow. Both parents were Jewish immigrants raised on the Lower East Side. Harry, a banker, was prosperous during the first decade of Kenneth's life, but when the Depression hit, Harry's bank failed, and the family spent the next decade poor.

When the family had money, they spent a lot of it on books, sets of the world's best literature and encyclopedias. Kenneth was such an

avid reader that his mother found it difficult to punish him. When she sent him to his room for having been involved in some mischief, he would simply select an encyclopedia volume and settle down happily with it. Lillian learned that she had to discipline her son by forcing him to go outside and play.

Lillian and Hany, both staunch supporters of Franklin Delano Roosevelt, would discuss politics at the dinner table. The newly impoverished Arrows had to send their son to City College of New York, a school that offered a free education to New Yorkers. The school's faculty had strong Marxist leanings. At City College, Kenneth became fascinated with logic and statistics.

"There was a very famous logician called Alfred Tarski," Arrow explained. "He came to New York for a conference in late August 1939 and was caught here by the outbreak of war." Arrow and his classmates struggled to understand Tarski's idiosyncratic English. One of the words Tarski taught them was as obscure as his pronunciation; *intransitivity*. This idea would become the heart of Arrow's impossibility theorem.

The best way to explain intransitivity is to start with its opposite, *transitivity*. If Bill Gates is richer than Donald Trump, and Donald Trump is richer than you, then it follows that Bill Gates is richer than you. Any relationship that permits such a conclusion is said to be transitive. Many other types of comparisons qualify-"heavier than," "taller than," "is the sister of." Many mathematical relationships are transitive, too. "Greater than," "less than," and "is equal to" are examples. When quantity A equals B, and B equals C, then A has to equal C as well.

An intransitive relationship is anything that does not follow this neat pattern. There are plenty of examples, and we never give most of them a second thought. Raymond is the son of John, and Keith is the son of Raymond. It obviously does *not* follow that Keith is the son of John. Lucas loves Margo and Margo loves Chris. It does not follow that Lucas loves Chris. These are intransitive relationships.

As much as Arrow enjoyed his excursions into arcane logic, "I never thought of this as a way to make a living. This was the Great Depression. The only thing I could dream of doing in mathematics was teaching in high school. I would have been very happy with that; the only problem was, there were no jobs."

The bleak employment picture sent Arrow to graduate work at Columbia University. There he decided that his primary interest was statistics. Harold Hotelling, a statistician in the economics department, was able to offer Arrow a fellowship on the condition that he switch his major to economics.

Hotelling's interests were diverse. In 1929 he proposed a famous riddle of economics, one that is equally important to political theory. There are two "places of business" located "along a line ... which may be Main Street in a town or a transcontinental railroad," Hotelling wrote. Or, as it's often explained today, the places of business are two ice-cream stands on a crowded summer beach. Where should each stand be located in order to get the most business?

The beach is, say, a thousand yards long, running left to right. The only difference between one stand and the other is location. Beachgoers will naturally favor whichever stand is closest.

One possible arrangement is to have the stands at the 250-yard and 750-yard positions (measured from the far left end of the beach). Then the stand at the 250-yard point will be closest to everyone on the left half of the beach (from 0 to 500 yards). That stand can expect half the ice-cream sales, assuming the customers are evenly scattered over the beach's length. The other stand would get the other half of the business.

This is *not* the answer to Hotelling's puzzle. Here's why. Say you put your stand at 250 yards, and the other vendor is at 750 yards. What's to prevent your competitor from moving in your territory? He could relocate his stand to the 300-yard point. In so doing, he would retain his lock on the whole right half of the beach. (His customers would have to walk farther, hut what choice do they have?) At the new location, he

would be the closest stand for anyone from the 275-yard point all the way to the I,OOO-yard limit. This would give him the lion's share of the business.

You wouldn't have to put up with that. You could leapfrog over him and steal most of his territory. He could counter-retaliate ... Is there any way of arriving at a sensible truce, where both vendors are satisfied that they could do no better by moving?

Hotelling's answer was yes. The optimal solution is for both vendors to be side by side in the exact middle of the beach. One vendor is just to the left of the precise midpoint and thereby commands the entire left half of the beach. The other vendor is a few inches right of the midpoint and wins the right half.

You may find this answer surprising. Hotelling was *not* saying that this solution was best for the customers. The customers at the far ends of the beach will have a long hike. Hotelling was simply saying this is the way a laissez-faire economy works. Provided no government regulation mandates distance between the stands, they will have incentive to move to the middle.

Many economists believe that Hotelling's model-the "tendency of the outermost entrepreneurs to approach the cluster"--explains some of the minor mysteries of the real world. Why are so many Starbucks located just across the street from *other* upscale coffee shops? Why do all SUVs look alike? Why do TV stations run the news at the same time? Why are the two most popular soft drinks both fizzy brown syrups that taste the same? And why do airlines schedule popular flights to leave within minutes of each other? The answer could be that vendors are competing for the same finite stretch of market "territory." Moving too far from the center would cede too much of the business to the competition. "Buyers are confronted everywhere with an excessive sameness," Hotelling wrote. "Methodist and Presbyterian churches are too much alike; cider is too homogeneous."

HoteHing was well aware that the same principle applies in politics. America's two major parties compete for voters who fall along a left-toright ideological spectrum. A voter normally favors the candidate whose ideology is closest to her own. Hence candidates of both parties have reason to gravitate toward the middle, to court the swing voters. "Each candidate 'pussyfoots,'" wrote Hotelling, "replies ambiguously to questions, refuses to take a definite stand in any controversy for fear of losing votes."

This model goes only so far. America's two parties *aren't* identical and aren't precisely in the middle. One likely reason is that citizens have the option of not voting and donors have the option of not contributing. If you want ice cream, you've got to hike to the nearest stand. But if the two candidates for office are identical, there is little reason to care who wins.

Arrow absorbed economics quickly. He completed all the required doctoral courses by 1942 and came to the slightly terrifying realization that he had no idea what to do for his dissertation. He thus began a long career as "professional student" *(ein Student, Oskar Morgenstern might have said).*

For a while he drifted from job to job. Then the war intervened, and he worked in weather prediction for the army. This convinced him that the physical sciences were not necessarily more accurate than the social sciences. He thought about chucking an academic career and becoming an actuary for an insurance company. He had heard there was good money in that.

Then Hotelling recommended him for an appointment at the Cowles Commission, the famous economic institute in Chicago. At Cowles, Arrow met many of the great economists of his time. Despite his lack of a Ph.D., he took on his first academic appointment (at the University of Chicago) and a more lasting appointment as husband to Selma Schweitzer. Schweitzer was herself a fellow at Cowles. They married on August 31, 1947. Meanwhile, Arrow continued his search for a dissertation topic.

"John Hicks gave a lecture at Columbia in 1946," Arrow recalled. "He had wanted a definition: What do you mean by saying individual A is better off than individual B?" This deceptively simple question was a vexing problem for economists. Who's better off, a fifty-dollar-an-hour oil field worker compelled to live on a rig in the North Sea, or a worker living a more normal life in Houston, at half the salary? How do you compare the overall welfare of people in a rat-race technological society with those in a more family-oriented third-world culture?

Hicks's tentative definition was that, in order for A to be better off than B, it is necessary that both A and B agree that A is better off. In effect, Hicks was proposing that the two people *vote* on who is better off. Only a unanimous vote would be considered decisive.

Arrow's hand shot up. 'What bothered me is that if you have a definition of 'better off: you'd like to be able to say that if A is better off than B and B is better off than C, then A is better off than C. It does not foHow! I could think of examples right away!" Arrow was talking about intransitivity. Hicks had no idea what he was talking about.

"A year later, I'm working on my thesis," Arrow continued. "I'm a great admirer of Hicks's book *Value and Capital* [1939]. But I could see, being the empirical character I am, some problems. I thought my thesis would be fixing them up."

One problem was how corporate stockholders vote on a new director. Provided there are three or more candidates, Arrow realized, it is possible for the results of voting to be intransitive. He devised a simple example, now called the "paradox of voting" or the "Arrow paradox."

Imagine the election is between three candidates whom I'll call Scissors, Paper, and Stone. The voters fall into three factions, each comprising a third of the total vote. One faction likes Scissors the best, Paper second best, and Stone the least. The second faction likes Paper best, Stone second, and Scissors last. The third faction likes Stone best, Scissors second, and Paper last.

Game Theory

	FirsT choice	Second choice	Third choice
FacTion A	Scissors	Paper	Stone
Faction B	Paper	Stone	Scissors
Faction C	Stone	Scissors	Paper

This creates a bizarre paradox. *Every* candidate can win a two-way vote. Every candidate can also *lose* a two-way vote.

Take a vote between Scissors and Paper. Factions A and C will prefer Scissors to Paper. Therefore Scissors will beat Paper by a two-thirds majority.

In a similar vote between Paper and Stone, Paper will win, and also by a two-thirds majority.

Now, if Scissors beats Paper and Paper beats Stone, it stands to reason that Scissors will beat Stone. *It doesn't.* In a vote between Scissors and Stone, Stone would win by a two-thirds majority. The outcome is just like the schoolyard game of the same name. Scissors beats Paper, Paper beats Stone, *and* Stone beats Scissors.

This notion strikes almost everyone as illogical, like an M. C. Escher picture of an endless waterfall or staircase. It is a blow to learn that voting, the very bedrock of free societies, is subject to this wild illogic.

The paradox of voting bedeviled Arrow's doctoral work. "Instead of seeing this as an intellectual opportunity, I thought, This is a nuisance." Arrow said. He naturally thought it presented an insurmountable roadblock to any reasonable model of corporate behavior (he was still focused on stockholder voting). Arrow also had a nagging sense of $d\acute{e}j$ à vu. "I thought 1 had heard it somewhere before," he said of the paradox. "From that day to this I have not been able to establish whether I had really heard it or not." This belief that the idea might not be original discouraged him from pursuing it. (Indeed, the paradox had been described by the Marquis de Condorcet in the eighteenth century but had been long forgotten.)

"There was a third episode, I got to think, what happens if parties are arrayed left to right?" Arrow applied the paradox to Hotelling's model of politics, He realized that when people's political views fall on a linear, liberal-to-conservative spectrum, the paradox of voting cannot occur.

Let the three candidates be Ralph Nader, Al Gore, and George W, Bush. There are six possible ways of ranking the three candidates, though not all of them make political sense. Whatever your politics, everyone agrees that Nader is on the far left, Gore is a little left of center, and Bush is somewhere to the right of center.

In order to have the paradox, you'd need about a third of the voters to prefer Nader to Gore and Gore to Bush. Okay, Then you'd need another large faction to prefer Gore to Bush and Bush to Nader. That's possible. Finally, you'd need a third faction to prefer Bush to Nader and Nader to Gore. That doesn't compute. It's hard to imagine that Bush supporters would sincerely prefer Nader over Gore.

Arrow sketched a proof that linear ideology prevents a scissorpaper-stone-type voting paradox. *This* insight struck him as unquestionably original and worthy of publication.

"I remember showing this to someone at lunch," Arrow remembered. "Then I picked up a copy of the *Journal of Political Economy* and there was Duncan Black with exactly that idea!"

Duncan Black was Arrow's only rival in founding a modern science of voting. Black taught at the University of Glasgow, far from the academic networks that counted, and lived in a house perched perilously on a cliff. No one at Glasgow quite knew what to make of Black's interest in voting. ("It had been brought to my attention," Black wrote, "that my main effort during the preceding years had produced no tangible result.")

The article that upstaged Arrow had its origin in the war, when Black was watching for air raids from the Green Drawing Room of Warwick Castle. "Acting apparently at random, I wrote down a single diagram and saw in a shock of recognition the property of the median optimum."

This was Black's *median voter theorem*. The "median voter" is the voter in the exact middle. Half the voters are more conservative than the median voter, and half are more liberaL

In order for there to be a median voter, it's necessary that everyone's political views be assignable to a point on a neat left-to-right line. That mayor may not be a good approximation to reality. But when there is a median voter, votes between pairs of candidates will be transitive, and Arrow's disturbing paradox can't occur. Black's theorem demonstrated that the median voter is the ultimate focus group. Whichever of two candidates the median voter prefers will win a two-way vote.

This seconds the conventional wisdom of politics. Pollster Richard Scammon and strategist Ben Wattenberg once semi-seriously remarked that the ultimate bellwether is a "forty-seven-year-old wife of a machinist living in Dayton, Ohio." The presidential candidate who captures *her* vote will win the election. There is nothing too mysterious about this. The median voter, like everyone else, favors the candidate whose views are closest to her own. This means that the candidate who captures the center will win a two-way race.

Six years Into his peripatetic career as grad student, Arrow accepted an unusual job. He agreed to go to California to think about nuclear doomsday.

The RAND Corporation was the greatest monument to von Neumann's-and Morgenstern's-game theory. RAND began as the air force's Project RAND (for Research ANd Development), a scientific consultancy initially contracted to Douglas Aircraft. Conceived as a peacetime Manhattan Project, RAND was recruiting many of the nation's best minds to ponder the challenges of the nuclear age.

Arrow heard about RAND from his wife's former employer, Abe Girschick. 'This Air Force thing at that point was a wild, far-out place, open to all kinds of ideas," Arrow said. 'The idea was that because of the new nature of warfare, particularly the bomb, all the old views were wrong ... It was an invitation to take a very wild point of view."

RAND took pride in hiring a diverse group of specialists and encouraging everyone to talk to one another. Over the years, RAND's scholars and consultants have ranged from John Nash to Condoleezza Rice. In its first decade, however, the guiding spirit of the place was unquestionably John von Neumann.

"Everyone sat up in great awe" when von Neumann spoke, Arrow said. Politically, von Neumann was conservative and a hawk. He believed that game theory provided useful models for nuclear deterrence and arms races. RAND's people pondered questions such as would the Soviet Union launch a first strike against the United States if it meant losing twenty million people in the counterattack? Would building a hydrogen bomb enhance or diminish U.S. security?

Arrow's title was research statistician and mathematician. He was asked to analyze the deployment of America's nuclear submarines. The submarines were constantly in motion so that the Soviets would never know where they all were at any given time and would thus be unable to destroy all the subs in a surprise first strike. Knowing that some American submarines would be able to launch a counterattack against the Soviet Union-and knowing that the Americans knew the Soviets knew this-were cornerstones of the policy of deterrence.

Arrow now questions how much of this work ended up being useful to American defense. "People were *trying* to be helpful," he said. "But really, we were making up our military problems by talking to each other." Many of RAND's most influential contributions far transcended the defense issues that inspired them. The impossibility theorem is one of the best examples of that.

"It was in this context that Helmer, one day at coffee-they had terrible coffee-said the United States after all is an *abstraction*. There are lots of people in the United States. They have different interests, different political values. So even assuming you had a value structure for every individual, in what sense do you have a value structure for the United States?"

Olaf Helmer was a philosopher. RAND was broad-minded enough to suppose that philosophers might have useful input in defense strategizing. Helmer's point was that game theory assumes the participation of "players" with precisely defined motivations. How could President Harry Truman or General Secretary Joseph Stalin speak and act for their nations?

Truman at least had been democratically elected. But how could he represent the people who had voted against him, or the people who had voted for him only because they thought Thomas Dewey was even worse? There were American hawks who couldn't build H-bombs fast enough. There were American pacifists clamoring for immediate unilateral disarmament. For almost anything that some American believes, there is another American who believes the exact opposite, The same was true of the Soviet Union, even if its people could not dissent openly.

Arrow's off-the-cuff response to Helmer was "Oh! That is nothing, Abram Bergson has written on that type of thing." Bergson, then at Columbia, was an expert on the Soviet economy, This was a tricky maUer, for the Soviet gross national product could not be measured by market valuation, as with capitalist economies, Bergson devised the concept of a "social welfare function," a numerical measure of a society's wellbeing. He had not fully addressed how societies make choices, however. Helmer suggested that Arrow write something on the subject.

"I just started playing around," Arrow said modestly. "It took me about two days to decide I was on the wrong track because 1 was looking for some solution. It didn't occur to me that there was no solution."

Arrow's result, the impossibility theorem, shows that indeed there is no solution for some of the problems of voting. The theorem was published first as a RAND research report in the fall of 1948. Almost



Walt Kelly's 1948 cartoon depicts the metaphor that inspired Kenneth Arrow: cold war as chess game between Harry Truman and Joseph Stalin, When democrat and dictator represent millions, who moves? (© Okefenokee Glee & Perloo, Inc. Used by permission; Collection of the Harry S Truman Library)

immediately it was a sensation. It became Arrow's long-delayed dissertation topic and was republished as a 195 1 book, *Social Choice and Individual Values*.

The theorem was so original that its first reactions included puzzlement and incomprehension. Columbia University economist Al Hart was charged with reviewing Arrow's dissertation, Theodore Anderson remembers Hart coming into his office and saying, "Ted, would you look at this? Don't tell me if it is correct, tell me if it is important."